1. The diagram below shows the line PQ, whose equation is $x + 2y = 12$. The line intercepts the axes at P and Q respectively.

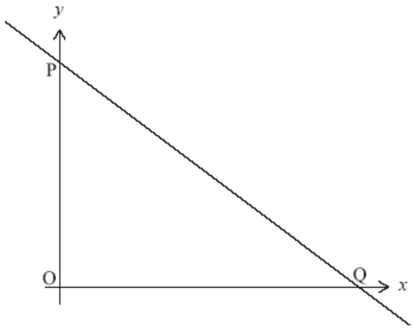


diagram not to scale

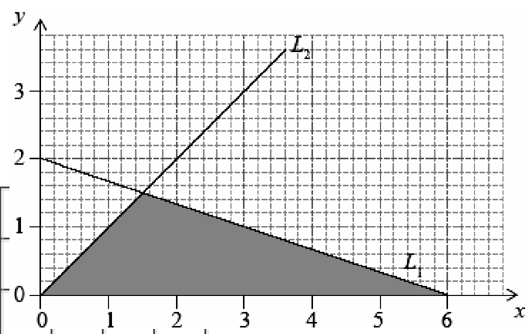
(a) Find the coordinates of P and of Q.

(3)

(b) A second line with equation $x - y = 3$ intersects the line PQ at the point A. Find the coordinates of A.

(3)
(Total 6 marks)

2. El gráfico siguiente muestra dos rectas L_1 y L_2. La ecuación de la recta L_2 es $y = x$.



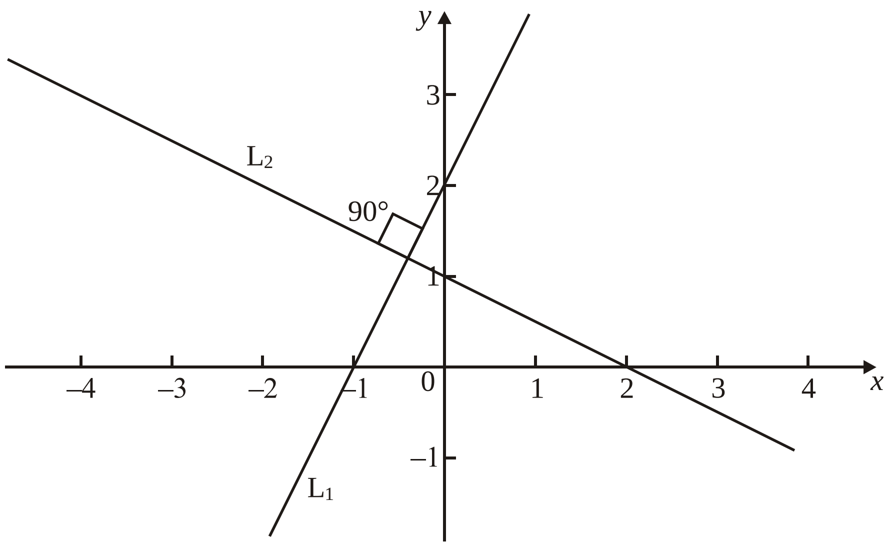
a) **Encuentre** el gradiente de L_1 y de L_2. [3 puntos]

b) **Escriba** la ecuación de L_1. [1 puntos]

c) **Encuentre** el punto donde se intersectan las dos rectas, utilizando el método de sustitución. [4 puntos]

[8 puntos total]

3. A student has drawn the two straight line graphs L_1 and L_2 and marked in the angle between them as a right angle, as shown below. The student has drawn one of the lines incorrectly.



Consider L_1 with equation $y = 2x + 2$ and L_2 with equation $y = -\frac{1}{4}x + 1$.

(a) Write down the gradients of L_1 and L_2 **using the given equations**.

(b) Which of the two lines has the student drawn incorrectly?

(c) How can you tell from the answer to part (a) that the angle between L_1 and L_2 should not be 90°?

(d) Draw the correct version of the incorrectly drawn line on the diagram.

(Total 8 marks)

4. La recta L_1 tiene por ecuación y = 3x + 4.

a) **Escriba** el gradiente de L_1. [1 punto]

b) Dos rectas son perpendiculares si el producto de sus gradientes es igual a – 1. **Demuestre** que el gradiente de la recta que es perpendicular a L_1 es $-\frac{1}{3}$. [1 puntos]

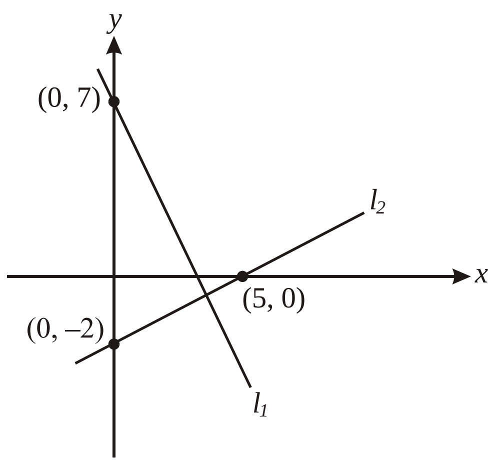
c) **Encuentre** la ecuación de la recta perpendicular a L_1 y que pasa por el punto (6, 7). Exprese su respuesta de tal manera que no hayan fracciones. [3 puntos]

d) **Calcule** las coordenadas del punto de intersección de las dos rectas, utilizando el método de eliminación. [4 puntos].

[9 puntos total]

5. The following diagram shows the lines l_1 and l_2, which are perpendicular to each other.

Diagram not to scale



(a) Calculate the gradient of line l_1.

(b) Write the equation of line l_1 in the form $ax + by + d = 0$ where a, b and d are integers, and $a > 0$.

(Total 8 marks)

6. Un carpintero elabora sillas y mesas de madera. Le toma 10 horas para elaborar una mesa y 4 horas para elaborar una silla. Gasta \$ 40 en materiales para elaborar cada silla, y \$ 120 para elaborar cada mesa. Si en un contrato el carpintero gasta \$ 760, y le toma terminar el trabajo 70 horas, encuentre el número de sillas y el número de mesas que elaboró. [4 puntos]

7. The conversion formula for temperature from the Fahrenheit (F) to the Celsius (C) scale is given by $C = \frac{5}{9}F - \frac{160}{9}$.

(a) What is the temperature in degrees Celsius when it is 50° Fahrenheit?

There is another temperature scale called the Kelvin (K) scale.
The temperature in degrees Kelvin is given by $K = C + 273$.

(b) What is the temperature in **Fahrenheit** when it is zero degrees on the Kelvin scale?

(Total 8 marks)

8. At Jumbo's Burger Bar, Jumbo burgers cost £J each and regular cokes cost £C each. Two Jumbo burgers and three regular cokes cost £5.95.

(a) Write an equation to show this.

(b) If one Jumbo Burger costs £2.15, what is the cost, in pence, of one regular coke?

(Total 4 marks)

9. El departamento de Matemáticas de un colegio tiene un presupuesto de $ 1440 para comprar libros de texto. El libro "Matemáticas para todos, volumen 1" tiene un costo de $ 70, mientras que el libro "Matemáticas para todos, volumen 2" tiene un costo de $ 40. Si el departamento necesita el doble de textos del volumen 1 que del volumen 2, encuentre cuántos libros comprará el departamento de Matemáticas en total. [4 puntos]

LOS SIGUIENTES EJERCICIOS SE RELACIONAN CON LOS MÉTODOS DE SOLUCIÓN DE SISTEMAS DE ECUACIONES LINEALES.NO SE CONSIDERAN COMO TAREA. REVÍSELOS Y REPÁSELOS

G LINEAR SIMULTANEOUS EQUATIONS

Simultaneous equations are two equations containing two unknowns, e.g., $\begin{cases} x+y=9 \\ 2x+3y=21. \end{cases}$

When we solve these problems we are trying to find the solution which is common to both equations.

Notice that if $x=6$ and $y=3$ then:

- $x+y=6+3=9$ ✓ i.e., the equation is satisfied
- $2x+3y=2\times 6+3\times 3=12+9=21$ ✓ i.e., the equation is satisfied.

So, $x=6$ and $y=3$ is the **solution** to the simultaneous equations $\begin{cases} x+y=9 \\ 2x+3y=21. \end{cases}$

The solutions to **linear simultaneous equations** can be found by **trial and error** (a little tedious) or **graphically** (which can be inaccurate if solutions are not integers).

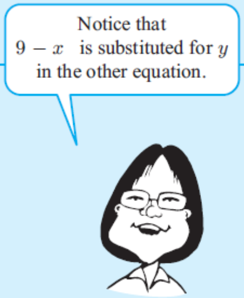
However, because of the limitations of these methods, other methods are used.

SOLUTION BY SUBSTITUTION

The method of **solution by substitution** is used when at least one equation is given with either x or y as the **subject** of the formula.

Example 20

Solve simultaneously, by substitution: $y=9-x$
$2x+3y=21$



$$y=9-x \quad \text{......(1)}$$
$$2x+3y=21 \quad \text{......(2)}$$

Since $y=9-x$, then
$$2x+3(9-x)=21$$
$$\therefore\ 2x+27-3x=21$$
$$\therefore\ 27-x=21$$
$$\therefore\ -x=21-27$$
$$\therefore\ -x=-6$$
$$\therefore\ x=6$$

and so, when $x=6$, $y=9-6$ {substituting $x=6$ into (1)}
$$\therefore\ y=3.$$

Solution is: $x=6$, $y=3$.

Check: (1) $3=9-6$ ✓ (2) $2(6)+3(3)=12+9=21$ ✓

Example 21

Solve simultaneously, by substitution: $2y-x=2$
$x=1+8y$

$$2y-x=2 \quad \text{...... (1)}$$
$$x=1+8y \quad \text{...... (2)}$$

Substituting (2) into (1) gives
$$\therefore\ 2y-(1+8y)=2$$
$$\therefore\ 2y-1-8y=2$$
$$\therefore\ -6y-1=2$$
$$\therefore\ -6y=3$$
$$\therefore\ y=-\tfrac{1}{2}.$$

Substituting $y=-\frac{1}{2}$ into (2) gives
$$x=1+8\times -\tfrac{1}{2}=-3.$$

The solution is $x=-3$, $y=-\frac{1}{2}$.

Check: (1) $2(-\frac{1}{2})-(-3)=-1+3=2$ ✓
(2) $1+8(-\frac{1}{2})=1-4=-3$ ✓

EXERCISE 6G.1

1 Solve simultaneously, using substitution:

a $x = 8 - 2y$
$2x + 3y = 13$

b $y = 4 + x$
$5x - 3y = 0$

c $x = -10 - 2y$
$3y - 2x = -22$

d $x = -1 + 2y$
$x = 9 - 2y$

e $3x - 2y = 8$
$x = 3y + 12$

f $x + 2y = 8$
$y = 7 - 2x$

2 Use the substitution method to solve simultaneously:

a $x = -1 - 2y$
$2x - 3y = 12$

b $y = 3 - 2x$
$y = 3x + 1$

c $x = 3y - 9$
$5x + 2y = 23$

d $y = 5x$
$7x - 2y = 3$

e $x = -2 - 3y$
$3x - 2y = -17$

f $3x - 5y = 26$
$y = 4x - 12$

3 a Use the method of substitution to try to solve the equations $y = 3x + 1$ and $y = 3x + 4$.

b What is the simultaneous solution for the equations in a?

4 a Use the method of substitution to try to solve the equations $y = 3x + 1$ and $2y = 6x + 2$.

b How many simultaneous solutions do the equations in a have?

SOLUTION BY ELIMINATION

In many problems which require the simultaneous solution of linear equations, each equation will be of the form $ax + by = c$. Solution by substitution is often tedious in such situations and the method of elimination of one of the variables is preferred.

In the method of **elimination**, we eliminate (remove) one of the variables by making the coefficients of x (or y) the **same size** but **opposite in sign** and then **adding** the equations. This has the effect of **eliminating** one of the variables.

Example 22

Solve simultaneously, by elimination: $4x + 3y = 2$ .... (1)
$x - 3y = 8$ .... (2)

We **sum** the LHS's and the RHS's to get an equation which contains x only.

$$\begin{array}{rl} & 4x + 3y = 2 \\ + & x - 3y = 8 \\ \hline & 5x \quad = 10 \quad \{\text{on adding the equations}\} \\ \therefore & x = 2 \quad \{\text{dividing both sides by 5}\} \end{array}$$

Let $x = 2$ in (1) $\therefore$ $4 \times 2 + 3y = 2$

$\therefore$ $8 + 3y = 2$

$\therefore$ $3y = 2 - 8$ {subtracting 8 from both sides}

$\therefore$ $3y = -6$

$\therefore$ $y = -2$ {dividing by 3 on both sides}

i.e., $x = 2$ and $y = -2$

Check: in (2): $(2) - 3(-2) = 2 + 6 = 8$ ✓

Respuestas de los ejercicios propuestos

EXERCISE 6G.1

1 a $x = 2,\ y = 3$ b $x = 6,\ y = 10$ c $x = 2,\ y = -6$
d $x = 4,\ y = 2\frac{1}{2}$ e $x = 0,\ y = -4$ f $x = 2,\ y = 3$

2 a $x = 3,\ y = -2$ b $x = \frac{2}{5},\ y = \frac{11}{5}$ c $x = 3,\ y = 4$
d $x = -1,\ y = -5$ e $x = -5,\ y = 1$ f $x = 2,\ y = -4$

3 a obtain $1 = 4$ b no solution

4 a obtain $2 = 2$ b an infinite number of solutions

Example 23

Solve simultaneously, by elimination: $3x + 2y = 7$
$2x - 5y = 11$

$3x + 2y = 7$ ......(1)
$2x - 5y = 11$ ......(2)

We can eliminate y by multiplying (1) by 5 and (2) by 2.

$$\begin{array}{rl} \therefore & 15x + 10y = 35 \\ + & 4x - 10y = 22 \\ \hline \therefore & 19x \quad = 57 \quad \{\text{on adding the equations}\} \\ \therefore & x = 3 \quad \{\text{dividing both sides by 19}\} \end{array}$$

Substituting $x = 3$ into equation (1) gives

$3(3) + 2y = 7$

$\therefore$ $9 + 2y = 7$

$\therefore$ $2y = -2$

$\therefore$ $y = -1$

So, the solution is: $x = 3,\ y = -1$. *Check:* $3(3) + 2(-1) = 9 - 2 = 7$ ✓
$2(3) - 5(-1) = 6 + 5 = 11$ ✓

Example 24

Solve by elimination: $3x + 4y = 14$
$4x + 5y = 17$

$$3x + 4y = 14 \quad \text{........(1)}$$
$$4x + 5y = 17 \quad \text{........(2)}$$

To eliminate x, multiply both sides of

$$\begin{array}{lrl} \text{(1) by 4:} & 12x + 16y = 56 & \text{.......(3)} \\ \text{(2) by } -3\text{:} & \underline{-12x - 15y = -51} & \text{......(4)} \\ & y = 5 & \{\text{on adding (3) and (4)}\} \end{array}$$

and substituting $y = 5$ into (2) gives

$$\begin{aligned} 4x + 5(5) &= 17 \\ \therefore \quad 4x + 25 &= 17 \\ \therefore \quad 4x &= -8 \\ \therefore \quad x &= -2 \end{aligned}$$

Check:
(1) $3(-2) + 4(5) = (-6) + 20 = 14$ ✓
(2) $4(-2) + 5(5) = (-8) + 25 = 17$ ✓

Thus $x = -2$ and $y = 5$.

Example 25

Two numbers have a sum of 45 and a difference of 13. Find the numbers.

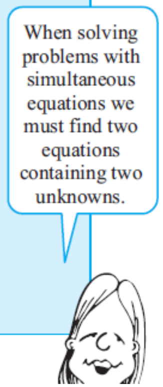
Let x and y be the unknown numbers, where $x > y$.

$$\begin{array}{lll} \text{Then} & x + y = 45 \quad \text{......(1)} & \{\text{`sum' means add}\} \\ \text{and} & \underline{x - y = 13} \quad \text{......(2)} & \{\text{`difference' means subtract}\} \\ & 2x = 58 & \{\text{adding (1) and (2)}\} \\ & \therefore \quad x = 29 & \{\text{dividing both sides by 2}\} \end{array}$$

and substituting into (1),
$29 + y = 45$
$\therefore \quad y = 16$

Check:
(1) $29 + 16 = 45$ ✓
(2) $29 - 16 = 13$ ✓

The numbers are 29 and 16.



WHAT TO ELIMINATE

There is always a choice whether to eliminate x or y, so our choice depends on which variable is easier to eliminate.

Solve the problem in **Example 24** by multiplying (1) by 5 and (2) by -4. This eliminates y rather than x. The final solutions should be the same.

2 Solve the following using the method of elimination:

a $4x - 3y = 6$, $-2x + 5y = 4$	**b** $2x - y = 9$, $x + 4y = 36$	**c** $3x + 4y = 6$, $x - 3y = -11$
d $2x + 3y = 7$, $3x - 2y = 4$	**e** $4x - 3y = 6$, $6x + 7y = 32$	**f** $7x - 3y = 29$, $3x + 4y + 14 = 0$
g $2x + 5y = 20$, $3x + 2y = 19$	**h** $3x - 2y = 10$, $4x + 3y = 19$	**i** $3x + 4y + 11 = 0$, $5x + 6y + 7 = 0$

3 The larger of two numbers is four times the smaller and their sum is 85. Find the two numbers.

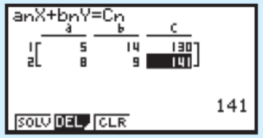
Example 26

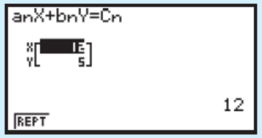
5 oranges and 14 bananas cost me \$1.30, and 8 oranges and 9 bananas cost \$1.41. Find the cost of each orange and each banana.

Let each orange cost x cents and each banana cost y cents. $\quad \therefore \quad 5x + 14y = 130 \text{ .... (1)}$, $\quad 8x + 9y = 141 \text{ .... (2)}$

{**Note:** Units must be the same on both sides of each equation i.e., cents}

From technology, $x = 12$, $y = 5$.





Thus oranges cost 12 cents, bananas cost 5 cents each.

Example 27

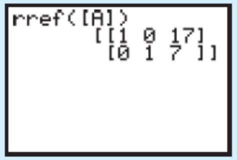
In my pocket I have only 5-cent and 10-cent coins. How many of each type of coin do I have if I have 24 coins altogether and their total value is \$1.55?

Let x be the number of 5-cent coins and y be the number of 10-cent coins.

$\therefore \quad x + y = 24 \quad$ ......(1) $\quad$ {the total number of coins}

and $\quad 5x + 10y = 155 \quad$ ......(2) $\quad$ {the total value of coins}

From technology, $\quad x = 17 \quad$ and $\quad y = 7$



Check: $\quad 17 + 7 = 24 \quad \checkmark$

$\qquad 5 \times 17 + 10 \times 7 = 85 + 70 = 155. \quad \checkmark$

Thus $\quad$ I have 17 five cent coins and 7 ten cent coins.

EXERCISE 6H

1 The sum of two numbers is 47 and their difference is 14. Find the numbers.

2 Find two numbers with sum 28 and half their difference 2.

3 The larger of two numbers is four times the smaller and their sum is 85. Find the two numbers.

4 Five pencils and 6 biros cost a total of \$4.64, whereas 7 pencils and 3 biros cost a total of \$3.58. Find the cost of each item.

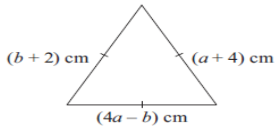
5 Seven toffees and three chocolates cost a total of \$1.68, whereas four toffees and five chocolates cost a total of \$1.65. Find the cost of each of the sweets.

6 I collect only 50-cent and \$1 coins. My collection consists of 43 coins and their total value is \$35. How many of each coin type do I have?

7 Amy and Michelle have \$29.40 between them and Amy's money is three quarters of Michelle's. How much money does each have?

8 Margarine is sold in either 250 g or 400 g packs. A supermarket manager ordered 19.6 kg of margarine and received 58 packs. How many of each type did the manager receive?

9 Given that the triangle alongside is equilateral, find a and b.



10 A rectangle has perimeter 32 cm. If 3 cm is taken from the length and added to the width, the rectangle becomes a square. Find the dimensions of the original rectangle.